NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM

No. 1179

DETERMINATION OF THE STRESS CONCENTRATION FACTOR OF A STEPPED SHAFT STRESSED IN TORSION BY MEANS OF PRECISION STRAIN GAGES

By A. Weigand

TRANSLATION

"Ermittlung der Formziffer der auf Verdrehung beanspruchten abgesetzten Welle mit Hilfe von Feindehnungsmessungen."
Luftfahrt-Forschung Band 20, Lieferung 7,
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DETERMINATION OF THE STRESS CONCENTRATION FACTOR OF A STEPPED SHAFT STRESSED IN TORSION BY MEANS OF PRECISION STRAIN GAGES*

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The stress distribution in stepped shafts stressed in torsion is determined by means of the electric precision strain gage by Lehr and Granacher [5]; the stress concentration factor α_k as a function of $\frac{d}{D}$ and $\frac{\rho}{D}$ for $0.5 \leqq \frac{d}{D} \leqq 0.9$ and $0.1 \leqq \frac{\rho}{d} < 0.25$ is ascertained from the measurements. It is shown that the test values always are slightly lower than the values resulting from an approximate formula by Sonntag [2].

Outline: I. Test Setup and Measuring Procedure

II. Evaluation of the Measurements and Results

III. Summary

IV. References

The stress distribution in stepped shafts stressed in torsion was first determined by F. A. Willers [1] by approximate integration of the differential equation of the stress function. R. Sonntag [2] gave an approximate formula for the stress function and the maximum stress for the case $D - d \geqq 2\rho$. (See fig. 1.) L. S. Jacobsen, A. Thum, and W. Bautz ([3] and the bibliography quoted there) ascertained the concentration factor by means of an electric model. The values found in this manner do not always agree with those calculated according to Sonntag; a discussion between these scientists ensued [4].

*"Ermittlung der Formziffer der auf Verdrehung beanspruchten abgesetzten Welle mit Hilfe von Feindehnungsmessungen." Zentrale für wissenschaftliches Berichtswesen der Luftfahrtforschung des Generalluftzeugmeisters, (ZWB) Berlin-Adlershof, Luftfahrt-Forschung Band 20, Lieferung 7, p. 217-219, München, July 20, 1943.

In order to definitely clear up this problem the stress distribution of steel shafts was determined by means of the precision strain gage developed by E. Lehr and H. Granacher [5].

I. TEST SETUP AND PROCEDURE

Figure 2 shows the apparatus which was used for twisting of the test shaft. Its modus operandi is explained in figure 3. The test shaft is obviously stressed by a pure torque. This fact was confirmed by a brittle lacquer test; the principal stress curves had an inclination of 45° toward the generatrices of the cylindric shaft.

The test points 1 to 11 for the strain measurements were arranged according to figure 5 on three generatrices I, II, and III which were 90° to each other. The two principal strains ϵ_1 and ϵ_2, which must be equal and opposite for pure torsion, were measured. Actually, the values deviate from the average usually by 3 to 4 percent; the reason is directional inaccuracy of the test section which was marked with a prick punch (1.3 mm) and a slight bending stress of the shaft caused by the fact that the twisting forces designated by α in figure 3 are not exactly equal. This error is essentially eliminated by measuring on three generatrices, figure 5. From the three test values which lie on the same circle the mean value is taken.

The strains were measured by means of the precision strain gage of 1.3-millimeter gage length developed by E. Lehr and H. Granacher; its modus operandi is described in detail in [5]. Before starting and at the end of each test series, which lasted three to four days per shaft, the instrument was calibrated on a tensile bar of known modulus of elasticity; the values found never differed by more than 1 to 2 percent and lay within the errors of measurement. It is true, the filament current for the small bulb of the strain gage must be kept exactly constant; at an intensity of the filament current of 120 mA fluctuations of 0.1 mA are already troublesome. Loose contacts, in particular, must be carefully avoided since irregular fluctuations would result in the multiflexgalvanometer measuring the photocurrent which has a sensitivity of about 4×10^{-6}mA per scale division.

For the tests an initial load of $P_0 = 50$ kilograms was selected and the deflection of the multiflexgalvanometer measuring the photo current in dependence on the loading was measured. This dependence is linear within the errors of measurement as can be seen in figure 6. The galvanometer delection was referred to 400-kilogram load increment; in figure 6 on the average 53 scale sectors correspond to this load

increment. Based on the (straight lined) calibration curve one now knows what strain corresponds to this load increment.

II. EVALUATION OF THE MEASUREMENTS AND RESULTS

From the known relation

$$\left.\begin{aligned} \sigma_1 &= \frac{E}{1 - \mu^2} (\epsilon_1 + \mu\epsilon_2) \\ \sigma_2 &= \frac{E}{1 - \mu^2} (\epsilon_2 + \mu\epsilon_1) \end{aligned}\right\} \tag{1}$$

between the principal strains ϵ_1, ϵ_2 and the principal stresses σ_1, σ_2 for the plane stress condition one obtains for the shear stress τ in the cross section of the shaft

$$\tau = \frac{\sigma_1 - \sigma_2}{2} = \frac{E}{1 + \mu} \frac{\epsilon_1 - \epsilon_2}{2} \tag{2}$$

The moduli of elasticity and of shear of the test shafts which were manufactured of St. C 45. 61., resulted as $E = 2.14 \times 10^6$, $G = 0.82 \times 10^6$ kg cm^{-2} and thus $\mu = \frac{1}{2}\frac{E}{G} - 1 = 0.305 \approx 0.30$.

Thus τ is for the present case

$$\tau = 0.823 (\epsilon_1 - \epsilon_2) \times 10^6$$

In figure 7 the distribution of shear stress along the shaft is represented for the case $\rho/d = 0.107$, $d/D = 0.70$. One can see that for the smooth part of the shaft the shear stress is constant except for slight scatter. The fillet begins at test point 8 shortly before the shear stress starts to increase and reaches its maximum at about test point 9 which lies already within the fillet. The resulting stress concentration factor for the present case is:

$$\alpha_k = \frac{\tau_{max}}{\tau_o} = \frac{715}{520} = 1.38$$

In the same manner the stress concentration factor was determined for a series of values of $\rho/d = \alpha$ and $d/D = \beta$. Table 1 shows the result. Fairing these test values graphically, one obtains the figures designated in table 1 as faired test values. These values are

plotted in figures 7 and 9 as functions of d/D and ρ/d. They are always somewhat lower than the values calculated from the approximate formula by Sonntag

$$\alpha_k = \beta(1.5 + 3.0\alpha)\ \frac{1 + 4\alpha}{1 + 6\alpha} + (1 - \beta - 2\alpha\beta)\left(1 + \frac{1}{12\alpha}\right) \qquad (3)$$

which is valid only for $\beta \leqq \frac{1}{1 + 2\alpha}$. Taking the unavoidable errors of measurement into consideration (for instance, inaccuracy of the punch mark, not ideal torsion) it will be permissible to assume the accuracy of these faired values with about 5 percent.

III. SUMMARY

The stress distribution in stepped shafts stressed in torsion was determined by means of percision strain gages. Comparison with an approximate formula set up by R. Sonntag showed that according to that formula the stress concentration factor α_k in the interval $0.1 \leqq \frac{\rho}{d} \leqq 0.25$ and $0.5 \leqq \frac{d}{D} \leqq 0.9$ can be calculated with sufficient accuracy.

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IV. REFERENCES

1. Willers, F. A.: Die Torsion eines Rotationskörpers um seine Achse. Z. Math. u. Phys. Bd. 55 (1907) p. 225.

2. Sonntag, R.: Zur Torsion von runden Wellen mit veränderlichem Durchmesser. Z. angew. Math. u. Mech. Bd. 9 (1929) p. 1.

3. Thum, A. and Bautz: Zur Frage der Formziffer. Z. VDI Bd. 79 (1935) p. 1303.

4. Sonntag, R., Thum, A., and Bautz, W.: Zur Frage der Formziffer. Z. VDI Bd. 81 (1937) p. 561.

5. Lehr, E., and Granacher, H.: Dehnungsmessgerät mit sehr kleiner Messtrecke und Anzeige mittels Sperrschicht-Photozelle. Forsch. a. d. Geb. d. Ingenieurwesens Bd. 7 (1936) p. 66.

TABLE I

THE STRESS CONCENTRATION FACTOR α_k AS A FUNCTION OF ρ/d AND d/D

ρ/d	d/D	α_k Measured	α_k Faired	α_k Calculated
0.25	0.50	1.17	1.22	1.23
	.667	1.18	1.19	1.20
	.824	1.21	1.13	----
	.90	1.13	1.09	----
.214	.50	1.24	1.25	1.27
	.70	1.22	1.20	1.22
	.824	1.19	1.19	----
	.90	1.13	1.10	----
.15	.50	1.33	1.36	1.37
	.70	1.22	1.28	1.30
	.80	1.20	1.23	----
	.90	1.13	1.15	----
.107	.50	1.43	1.45	1.49
	.60	1.43	1.41	1.44
	.70	1.38	1.35	1.38
	.80	1.26	1.30	1.32
	.90	1.19	1.22	----

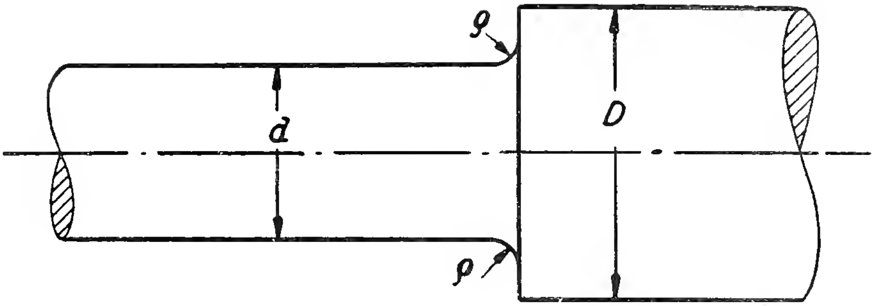

Figure 1.- Designations at the stepped shaft.

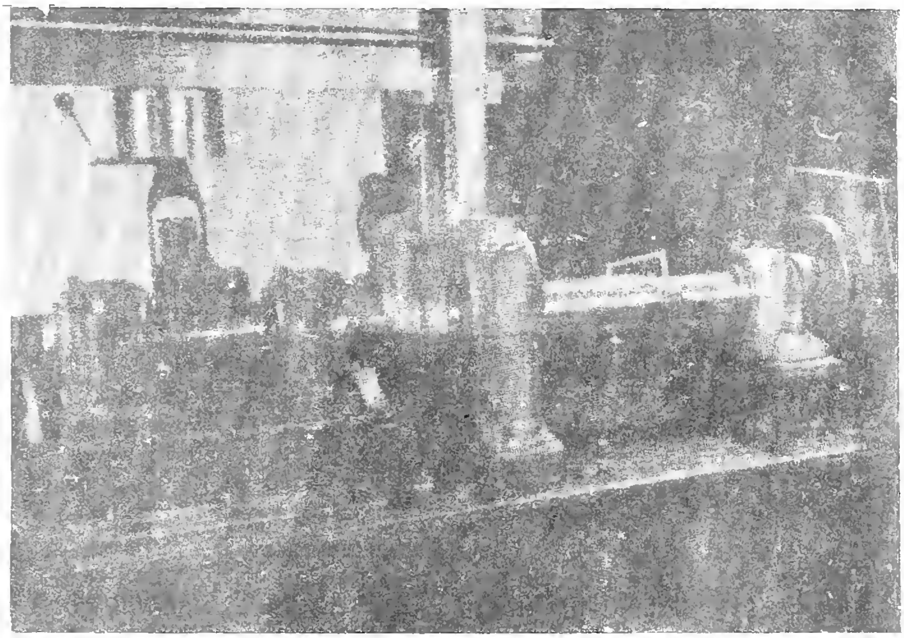

Figure 2.- Torsion machine.

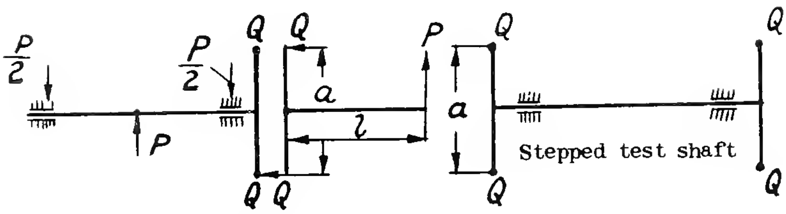

Figure 3.- Sketch explaining the operation method of the torsion machine.

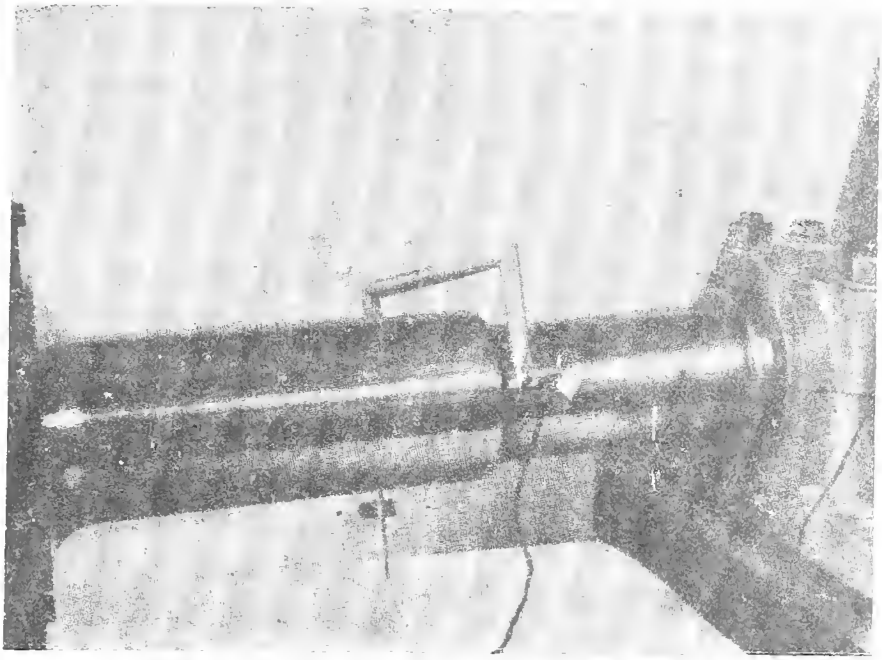

Figure 4.- Test shaft with strain gauge.

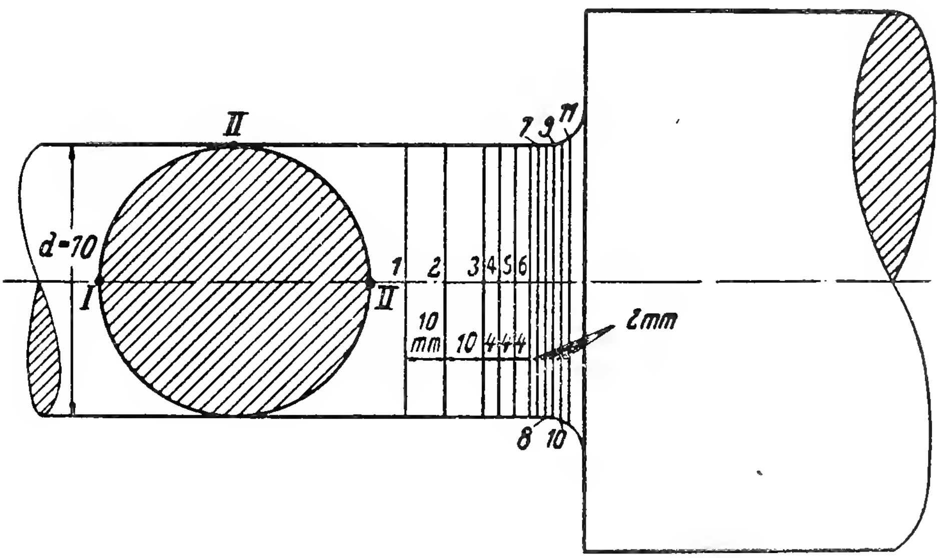

Figure 5.- Arrangement of the test points 1 to 11 on the generatrices I to III of the test sh. t.

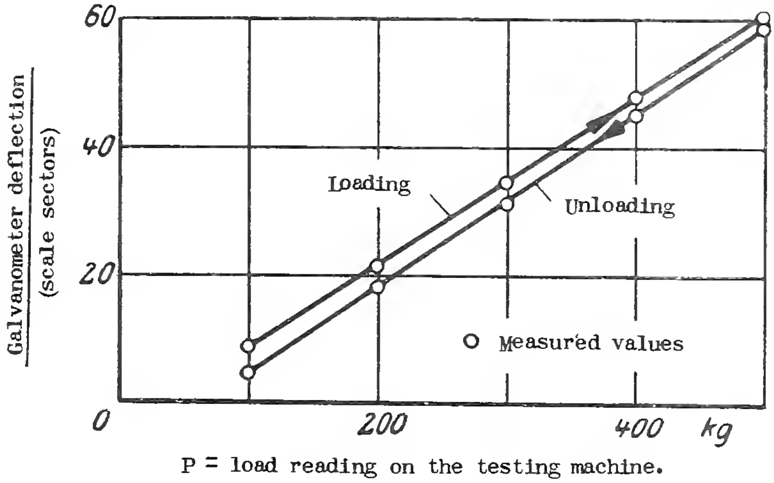

Figure 6.- Dependence of the photo current (galvanometer - deflection) on the load reading of the testing machine for a certain test point.

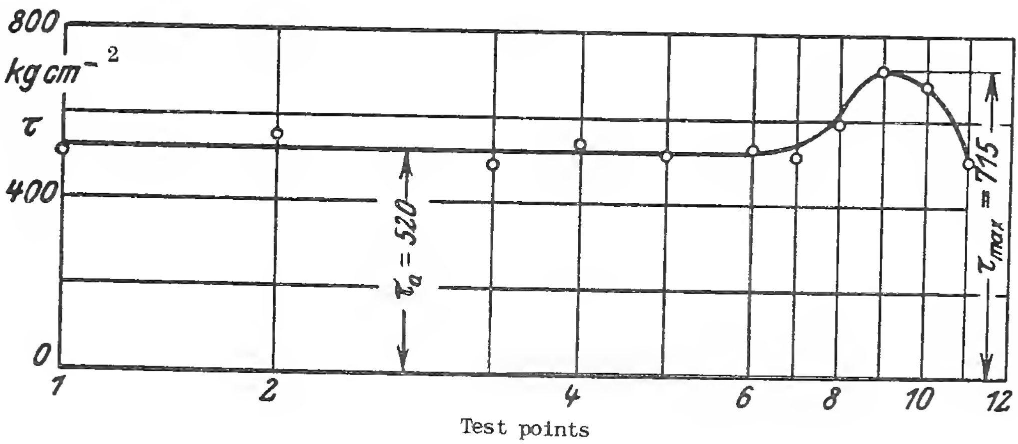

Figure 7.- Distribution of the shear stress along a generatrix of the stepped shaft for $\rho/d = 0.107$, $d/D = 0.70$.

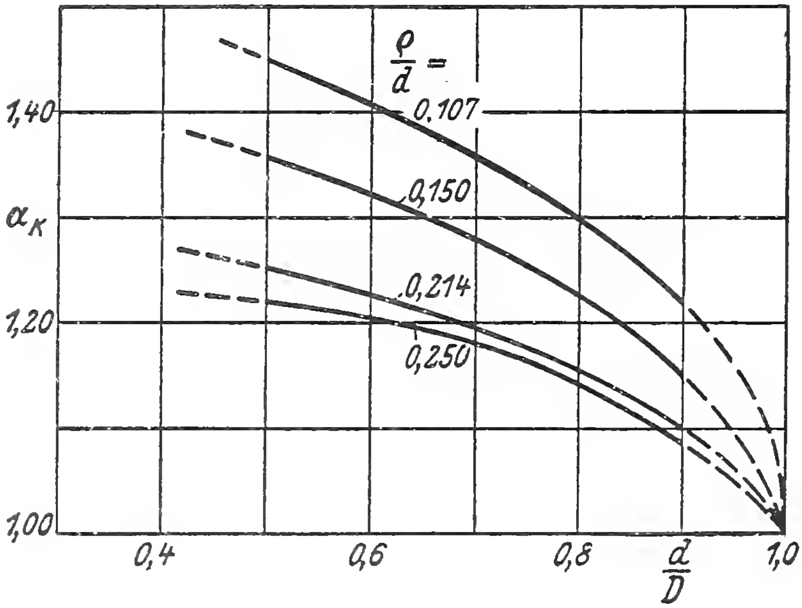

Figure 8.- The concentration factor α_k as a function of d/D.

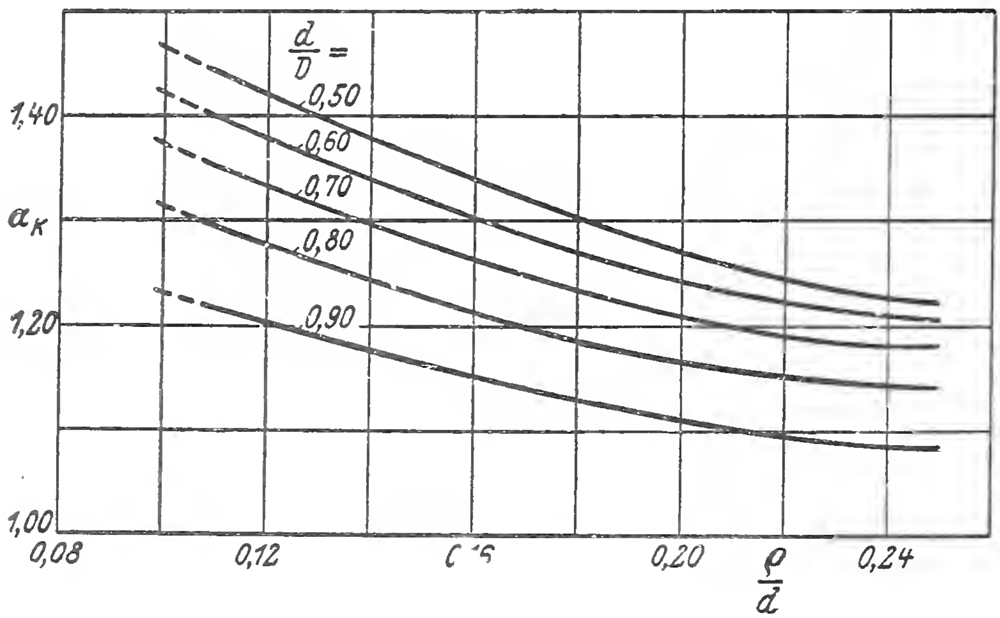

Figure 9.- The concentration factor α_k as a function of ρ/d.